Algorithm for Solving of Two-level Hierarchical Minimax Program Control Problem of Final State the Regional Socio-economic System in the Presence of Risks

A. F. Shorikov[a)]

Ural Federal University, 19 S. Mira, 620002 Ekaterinburg, Russia

[a)]afshorikov@mail.ru

Abstract. In this paper we study the problem of optimization of guaranteed result for program control by the final state of regional social and economic system in the presence of risks. For this problem we propose a mathematical model in the form of two-level hierarchical minimax program control problem of the final state of this process with incomplete information. For solving of its problem we constructed the common algorithm that has a form of a recurrent procedure of solving a linear programming and a finite optimization problems.

INTRODUCTION

In this paper we study the problem of optimization of guaranteed result for program control by the final state of regional social and economic system in the presence of risks. For mathematical modeling of this problem we consider a discrete-time dynamical process consisting of a set a controllable objects (region and forming it municipalities). The dynamics each of these is described by the corresponding linear discrete-time recurrent vector relations and its control system consist from two levels: basic level (the level *I*) that is dominant level and auxiliary level (the level *II*) that is subordinate level. Both control levels have different criterions of functioning and united by information and control connections which defined in advance. For this problem we propose a mathematical model in the form of two-level hierarchical minimax program control problem of the final state of this process with incomplete information. For solving of its problem we constructed the common algorithm that has a form of a recurrent procedure of solving a linear programming and a finite optimization problems. Results obtained in this paper are based on the studies [1]-[5] and can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in works [1]-[3], [6], [7].

DYNAMICS OF REGION SOCIAL AND ECONOMIC CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0, 1, \cdots, T\}$ $(T > 0,\ T \in \mathbb{N})$ we consider a controlled multistep dynamical process which consists of the $(n + 1)$ objects $(n \in \mathbb{N})$. Dynamics of the object I (main object of the system – region) controlled by dominant player P, is described by a vector linear discrete-time recurrent relation of the form

$$y(t + 1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)w(t),\ y(0) = y_0. \tag{1}$$

The dynamics of the object II_i (ith auxiliary object of the system – ith municipality) controlled by subordinate player E_i $(i \in \overline{1,n})$, is described by the following linear relation:

$$z^{(i)}(t + 1) = A^{(i)}(t)z^{(i)}(t) + B^{(i)}(t)u(t) + C^{(i)}(t)v^{(i)}(t) + D^{(i)}(t)w^{(i)}(t),\ z^{(i)}(0) = z_0^{(i)}, \tag{2}$$

where $t \in \overline{0,T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbb{R}^r$ is a vector of phase variables or phase vector of the object I – a set of main parameters describing the social and economic state of a region at the time moment t; $z^{(i)}(t) =$

Application of Mathematics in Technical and Natural Sciences
AIP Conf. Proc. 1895, 050012-1–050012-11; https://doi.org/10.1063/1.5007384
Published by AIP Publishing. 978-0-7354-1579-9/$30.00

(note, that the sets $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),\tilde{u}^{(e)}(\cdot))$ and $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},g(\tau))$ constructed from relations (20) and (24), respectively, and then the solution of this problem is reduced to solving the finite discrete optimization problem);

2) for any control $\tilde{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T})$ of the player P and any control $\bar{v}^{(i,e)}(\cdot)\in\bar{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))$ of the player E_i the constructing of the number $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))$ from solving the finite discrete optimization problem described by the relation (19) and satisfying the following relation:

$$\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))=\kappa^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(i,e)}(\cdot))$$

$$=<e^{(i)},\bar{z}^{(i,e)}(T)>_{s-i}=\max_{z^{(i)}(T)\in\mathbf{Z}^{(i)}(\tau,z^{(i)}(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(i,e)}(\cdot),Z_1(\cdot),T)}<e^{(i)},z^{(i)}(T)>_{s_i}. \quad (27)$$

Taking into consideration (18)-(27), and the conditions stipulated for the discrete-time dynamical process (1)-(8), one can prove that the following assertion is valid.

Theorem 3. For fixed time interval $\overline{\tau,T}\subseteq\overline{0,T}$ $(\tau<T)$ and admissible on the level I of the two-level hierarchical control system for the discrete-time dynamical process (1)-(8) realization the τ-position $g(\tau)=\{\tau,y(\tau),z^{(1)}(\tau),z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\}\in\mathbf{G}(\tau)$ $(g(0)=\{0,y_0,z^{(1)}_0,z^{(2)}_0,\cdots,z^{(n)}_0\}=g_0\in\hat{\mathbf{W}}_0)$ of the player P and admissible on the level II of the control system for this dynamical process the realization τ-position $\hat{g}(\tau)\in\hat{\mathbf{G}}(\tau)$ $(\hat{g}(0)=\hat{g}_0\in\hat{\mathbf{G}}_0)$ of the player E which formed due from the τ-position $g(\tau)$ and admissible realization of the program minimax control $u^{(e)}(\cdot)\in\mathbf{U}^{(e)}(\overline{\tau,T},g(\tau))$ of the player P on the level I of the control system, which formed from solving the Problem 1 and Problem 2, the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))\subseteq\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))\subseteq\mathbf{V}(\overline{\tau,T};u^{(e)}(\cdot))$ of the admissible program controls $\bar{v}^{(e)}(\cdot)\in\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))$ of the player E on the level II of the control system for this dynamical process and the number $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(1)(\tau),u^{(e)}(\cdot))$ which form due from (26) and (27), respectively, constructed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problem, and the following equalities are true:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))=\hat{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot));\ \bar{c}^{(e)}_{\alpha}(\overline{\tau,T},g(\tau))=c^{(e)}_{\alpha}(\overline{\tau,T},g(\tau));$$

$$\forall\, i\in\overline{1,n}:\ \bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot))=c^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot)). \quad (28)$$

Then from this assertion follows that a solution of the Problem 3 for the discrete-time dynamical process (1)-(8) can be formed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problems on the basis of construction of the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))$ and numbers $\bar{c}^{(e)}_{\alpha}(\overline{\tau,T},g(\tau))$ and $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot))$.

Note, that on the basis of the above algorithms of solving the Problems 1-3 the procedure – common algorithm of the construction a solution of the main problem of two-level hierarchical minimax program control by the final phase states of the objects I and II_i, $i\in\overline{1,n}$ for the discrete-time dynamical process (1)-(8) in the presence of perturbations can be formed from realization of a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problems.

CONCLUSION

In conclusion we note that more concrete algorithm for realization of the minimax program terminal control by the final phases state of regional social and economic system in the presence of risks for the discrete-time dynamical process (1)-(8) can be constructed using algorithms for solving minimax program terminal control problems with incomplete information from works [3], [4].

Results of this paper can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty.

ACKNOWLEDGEMENTS

This work was supported by the Russian Science Foundation, project no. 15-18-10014, “Projection of optimal socio-economic systems in turbulence of external and internal environment.”