Problem of Two-Level Hierarchical Minimax Program Control the Final State of Regional Social and Economic System with Incomplete Information

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Abstract. In this article we consider a discrete-time dynamical system consisting of a set a controllable objects (region and forming it municipalities). The dynamics each of these is described by the corresponding linear or nonlinear discrete-time recurrent vector relations and its control system consist from two levels: basic level (control level *I*) that is dominating level and auxiliary level (control level *II*) that is subordinate level. Both levels have different criterions of functioning and united by information and control connections which defined in advance. In this article we study the problem of optimization of guaranteed result for program control by the final state of regional social and economic system in the presence of risks vectors. For this problem we propose a mathematical model in the form of two-level hierarchical minimax program control problem of the final states of this system with incomplete information and the general scheme for its solving.

INTRODUCTION

In this paper we consider a dynamical system consisting of a set a controllable objects. The phase vectors of object I and each object II_i $(i \in \overline{1,n})$ $n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers) are describe respectively a states of social and economic parameters of a region and i-th municipality is located in this region. We assume that in this system a dynamics of object I, controlled by the dominating player-pursuer P, and a dynamics of each object II_i $(i \in \overline{1,n})$, controlled by the subordinate player E_i, are given by the vector linear and nonlinear discrete-time recurrent relations respectively. The control system for this dynamical system has two control levels: dominating level (first level or level I) that is regional level and subordinate level (second level or level II) that is municipal level and both have different criterions of functioning and united a priori by determined informational and control connections defined in advance. In the interest of a dominant player P are final (terminal) values of phase states of all objects being examined: object I — the main one (the region at large) and objects II_i, $i \in \overline{1,n}$ — n subsidiary objects (municipalities being a part of a region), and several its quality criterions. While in the interest of each subordinate player E_i are only values of final phase vector of subsidiary object II_i and a corresponding of its quality criterion. It is also assumed that the choice of a control action on the control level II by a subordinate player E_i depends from the choice of a control action on the control level I by a dominant player P. Quality of control process for this dynamical system on each level of the control system is estimated by the corresponding convex functional that are determined in their terminal (final) phase vectors and satisfied corresponding Lipschitz condition. It is assumed that phase vectors of all objects, control actions and undetermined a priory risks vectors in this dynamical system at every instant are limited with given finite sets in corresponding finite-dimensional vector spaces or satisfy by corresponding systems of a linear number $c^{(e)}_{\hat{\beta}^{(i)}}(\overline{\tau, T}, g^{(i)}(\tau), u(\cdot))$ which is the a value of the result of the minimax program control for this player on the level II of this control process which corresponds to the control $u(\cdot)$ and both are satisfies to the relation (15); on the base of this elements and from solving of the n problems 1 for all values of index $i \in \overline{1, n}$ we form the set $\mathbf{V}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u(\cdot))$ of the minimax program controls of the player E on the level II of this control process and the vector $c^{(e)}_{\hat{\beta}}(\overline{\tau, T}, \hat{g}(\tau), u(\cdot))$, which is a value of the result of the minimax program control of the player E on the level II of the control for this control process;

2) from solving of the problem 2 are forming the set $\mathbf{U}^{(e)}(\overline{\tau, T}, g(\tau))$ of the minimax program controls of the player P on the level I of the control of this control process and the number $c^{(e)}_{\hat{\alpha}}(\overline{\tau, T}, g(\tau))$ which is a value of the result of the minimax program control for the player P on the level I of the control of this control process and satisfying the relation (16);

3) for any minimax program control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, T}, g(\tau))$ of the player P on the level I of the control of this control process from solving of the problem 3 are forming the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau, T}; u^{(e)}(\cdot))$ of the optimal minimax program controls $\hat{v}^{(e)}(\cdot) = \{\hat{v}^{(1,e)}(\cdot), \hat{v}^{(2,e)}(\cdot), \cdots, \hat{v}^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u^{(e)}(\cdot))$ of the player E on the level II of the control of this control process and vector $c^{(e)}_{\hat{\beta}}(\overline{\tau, T}, \hat{g}(\tau), u^{(e)}(\cdot)) = (c^{(e)}_{\hat{\beta}^{(1)}}(\overline{\tau, \mathrm{T}}, g^{(1)}(\tau), u^{(e)}(\cdot)), c^{(e)}_{\hat{\beta}^{(2)}}(\overline{\tau, \mathrm{T}}, g^{(2)}(\tau), u^{(e)}(\cdot)), \cdots, c^{(e)}_{\hat{\beta}^{(n)}}(\overline{\tau, \mathrm{T}}, g^{(n)}(\tau), u^{(e)}(\cdot))) \in \mathbf{E}^n$ of optimal value of the result of the minimax program control for the player E on the level II of the control of this control process for considered dynamical system and corresponding the control $u^{(e)}(\cdot)$ of the player P.

CONCLUSION

In conclusion we note that a concrete algorithm for realization of the minimax program terminal control by the final state of regional social and economic system in the presence of risks vectors for the discrete-time dynamical system (1)–(6) can be constructed on the base algorithms for solving minimax program terminal control problems with incomplete information from works [3] and [4].

Results of this report can be used for computer simulation, design and construction of multilevel control systems for actual economic dynamical processes operating under deficit of information and uncertainty.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, project no. 15-18-10014, ”Projection of optimal socio-economic systems in turbulence of external and internal environment”.

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